Kinetic Theory of the Thermomagnetic Force

S. Hess *

Department of Physics, University of Toronto, Canada

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Results obtained within the framework of the kinetic theory of polyatomic gases, are reported for the influence of a magnetic field on the thermal force exerted on a solid sphere in a heat conducting gas.

Recently Larchez and Adair[1] reported the first measurement of a "thermomagnetic force" (TMF), i. e., of the magnetic-field-induced change of the thermal force (radiometric force) exerted on an obstacle in a heat conducting polyatomic gas. The underlying physical process responsible for the TMF is essentially the same as that leading to an influence of a magnetic field on the heat conductivity and on the viscosity[2, 3] (Senftleben-Beenakker effect, SBE), and to the thermomagnetic torque[4].

This letter is concerned with the kinetic theory of the TMF and its relation to the SBE. In particular, the thermal force acting on a solid sphere with radius R is considered. Results are reported for the two limiting cases $R \ll l$ and $R \gg l$ where l is the mean free path of a molecule in the gas.

The TMF, $\boldsymbol{F}$, is defined by

$$\boldsymbol{F}^{\text{th}}(\boldsymbol{H}) = \boldsymbol{F}^{\text{th}}(\boldsymbol{H}=0) + \boldsymbol{F}\,, \tag{1}$$

where $\boldsymbol{F}^{\text{th}}(\boldsymbol{H})$ is the thermal force in the presence of the magnetic field $\boldsymbol{H} = H\,\boldsymbol{h}$ $(\boldsymbol{h}\cdot\boldsymbol{h} = 1)$. The thermal force is proportional to the temperature gradient ∇T far away from the obstacle. The force acting on a sphere can be written as $\boldsymbol{F}^{\text{th}}(\boldsymbol{H}=0) = G\,R\,T^{-1}\,\nabla T$, and

$$\boldsymbol{F} = G\,R\,T^{-1}[K_{\parallel}(\nabla T)_{\parallel} + K_{\perp}(\nabla T)_{\perp} + K_{\text{trans}}\,\boldsymbol{h}\times\nabla T]\,, \tag{2}$$

where G, $K_{\parallel}$, $K_{\perp}$, K_{trans} are scalar coefficients, and $(\nabla T)_{\parallel,\perp}$ are the components of ∇T which are parallel to and perpendicular to $\boldsymbol{H}$. The subscript "trans" refers to "transverse" since $K_{\text{trans}} \neq 0$ gives rise to a transverse force perpendicular to both $\boldsymbol{H}$ and ∇T. Theoretical expressions for the field-free thermal force (and thus for G) have been derived by Einstein[5] and by Waldmann[6] for $R \ll l$, and by Epstein[7, 8] for $R \gg l$.

The thermal force acting on a particle which is small compared with the mean free path results from the fact that the gas molecules impinging on it from opposite sides have different mean velocities[5, 6, 8]. A straightforward generalization of the calculation given in Ref.[6] for monatomic gases to the case of polyatomic gases shows that the TMF is determined by the magnetic-field-dependence of the translational part of the heat flux. The influence of a magnetic field on the translational heat conductivity can be investigated within the framework of the kinetic theory based on the generalized Boltzmann equation[9] (Waldmann-Snider equation) much as the previously studied magnetic-field-induced change of the heat conductivity and of the viscosity[3].

The thermal force acting on particles which are large compared with the mean free path is caused by the drag of a flow velocity which is set up by a tangential temperature gradient at the surface of the particle[7, 8]. It has been pointed out by Waldmann[10] that the thermal force as calculated by Epstein[7] is equivalent to the Stokes force acting on a particle at rest if the velocity of the fluid far away from the particle is formally put proportional to ∇T (with an appropriately chosen proportionality coefficient). Hence in this case the calculation of the TMF is reduced to the calculation of the magnetic-field-induced change of the Stokes force. If it is assumed that the boundary conditions and the velocity field are in first approximation not affected by the applied field the TMF is essentially determined by the magnetic-field dependence of the viscosity tensor[3, 11].

For both small and large spheres the results can be written as

$$K_{\parallel} = -B_i\,f(\varphi_i)\,,$$
$$K_{\perp} = -B_i\,\tfrac{1}{2}[2\,f(2\,\varphi_i) + f(\varphi_i)]\,, \tag{3}$$

$$K_{\text{trans}} = B_i\,\tfrac{1}{2}[2\,g(2\,\varphi_i) + g(\varphi_i)]\,, \tag{4}$$

with $f(x) = x^2(1+x^2)^{-1}$, $g(x) = x(1+x^2)^{-1}$. Here $i = 1, 2$ refers to $R \ll l$ and $R \gg l$, respectively. The angle over which the rotational angular momentum of a molecule with a gyromagnetic ratio γ precesses about the magnetic field (precession frequency $\omega_{\text{H}} = \gamma\,H$) during an effective time of free flight $\tau_i = \omega_i^{-1}$ is denoted by φ_i. If wall collisions can be ignored the relaxation frequency ω_i is related to the number density n of the gas by $\omega_i = n\;(16\,k\,T/\pi\,m)^{1/2}\,\sigma_i$ where m is the mass of a molecule and σ_i is an effective cross section. Thus one has

$$\varphi_i = \omega_H/\omega_i = (\gamma/4\,\sigma_i)\,(\pi\,m\,k\,T)^{1/2}\,H/P\,, \tag{5}$$

where $P = n\,k\,T$ is the pressure of the gas. Hence the TMF depends on H and P via H/P. According to (3) $K_{\parallel}$ and $K_{\perp}$ reach saturation values ($-B_i$ and $-3/2\,B_i$, respectively) for $H/P \to \infty$. Both the dependence on H/P and the saturation behaviour agree with the experimental observations reported in Ref.[1] where, however, the thermal force on a disc rather than on a sphere has been investigated. The transverse TMF, which has not yet been measured, vanishes for both $H/P = 0$ and $H/P \to \infty$ and reaches a maximum value in between. The direction of this transverse force depends on the sign of the gyromagnetic ratio.

In more detail, ω_1 is equal to the relaxation coefficient ω_{K} of the "Kagan vector"[3] $\langle \boldsymbol{p}\cdot\boldsymbol{J}\,\boldsymbol{J}\rangle$ and ω_2 is

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Reprint requests to Dr. S. Hess, Institut für Theoretische Physik, Erlangen.

* On leave of absence from the Institute for Theoretical Physics, University of Erlangen-Nürnberg, Erlangen, Germany.

equal to the relaxation coefficient ω_{T} of the "tensor polarization" $\langle \boldsymbol{J}\boldsymbol{J} \rangle$. Here $\boldsymbol{p}$ and $\boldsymbol{J}$ are the linear and angular momenta of a molecule. The coefficients ω_{K} and ω_{T} can be inferred from the SBE on heat conductivity and the viscosity[3], respectively. Both ω_{K} and ω_{T} occur also in connection with the collisional and diffusional broadening of the depolarized RAYLEIGH line[12, 13].

The quantities B_i (as well as ω_i) can be expressed in terms of collision integrals[11, 13] obtained from the linearized Waldmann-Snider collision term. Comparison with the corresponding expressions for the maximal relative magnetic-field-induced change of the viscosity $(\Delta\eta/\eta)_{\mathrm{sat}}$ and of the heat conductivity $(\Delta\lambda_{\|}/\lambda)_{\mathrm{sat}}$ ("sat" refers to "saturation value") yields the relations[14]

$$B_1 = \sqrt{2}/5(m\,\lambda/k\,\eta)^{1/2}[\omega_{\mathrm{T}}/\omega_{\mathrm{K}}(\Delta\lambda_{\|}/\lambda)_{\mathrm{sat}}\,(\Delta\eta/\eta)_{\mathrm{sat}}]^{1/2}, \quad (6)$$

$$B_2 = 2/5\,|(\Delta\eta/\eta)_{\mathrm{sat}}|, \quad (7)$$

where λ and η are the field-free heat conductivity and viscosity. Typically, $B_{1,2}$ are of the order of 10^{-3} to 10^{-2}. Notice that due to $B_i > 0$ the application of a magnetic field results (in accord with experimental findings reported in Ref. [1]) in a decrease of the thermal force.

The thermal force acting on a disc which is perpendicular to the temperature gradient is proportional to the translational heat flux if it is both very thin and short compared with the mean free path. Then the TMF for $\boldsymbol{H}$ parallel and perpendicular to ∇T is determined by the coefficients $K_{\|}$ and $K_{\perp}$ as given by Eq. (3) with $i=1$ (ratio of the saturation values: 2/3).

In the experiments described in Ref. [1] the disc was thin but not short compared with the mean free path. Thus deviations from the above mentioned simple formulas can be expected and are indeed observed. Under these circumstances the theoretical interpretation of the data, however, is rather complicated. For this reason, measurements of the TMF acting on a sphere would be preferable. Furthermore, to settle the question whether the H/P dependence of the TMF is similar to that of the SBE of the heat conductivity or of the viscosity, measurements should be made for HD where $\omega_1 \equiv \omega_{\mathrm{K}}$ and $\omega_2 \equiv \omega_{\mathrm{T}}$ differ by almost one order of magnitude.

Finally, it seems worth mentioning that – for polar gases – the thermal force can also be affected by an applied electric field. In this case, however, no transverse force exists.

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[1] M. E. LARCHEZ and T. W. ADAIR, Phys. Rev. Lett. **25**, 21 [1970]; Phys. Rev. **A 3**, 2052 [1971].

[2] H. SENFTLEBEN, Phys. Z. **31**, 822, 961 [1930]. – J. J. M. BEENAKKER et al., Phys. Lett. **2**, 5 [1962].

[3] For a review see J. J. M. BEENAKKER and F. R. MCCOURT, Ann. Rev. Phys. Chem. **21**, 47 [1970].

[4] G. G. SCOTT, H. W. STURNER, and R. M. WILLIAMSON, Phys. Rev. **158**, 117 [1967].

[5] A. EINSTEIN, Z. Phys. **27**, 1 [1924].

[6] L. WALDMANN, Z. Naturforsch. **14 a**, 589 [1959].

[7] P. S. EPSTEIN, Z. Phys. **54**, 537 [1929].

[8] The thermal force is proportional to R^2 and independent of the pressure for $R \ll l$; it is proportional to R and inversely proportional to the pressure for $R \gg l$; see also N. A. FUCHS, The mechanics of aerosols, Pergamon Press, Oxford 1964; L. WALDMANN and K. H. SCHMITT, in: Aerosol Science, ed. C. N. DAVIES, Academic Press, New York 1966.

[9] L. WALDMANN, Z. Naturforsch. **12 a**, 660 [1957]; **13 a**, 609 [1958]. – R. F. SNIDER, J. Chem. Phys. **32**, 1051 [1960].

[10] See the appendix of the article by K. H. SCHMITT and L. WALDMANN, Z. Naturforsch. **15 a**, 843 [1960].

[11] F. R. MCCOURT and R. F. SNIDER, J. Chem. Phys. **47**, 4117 [1967]. – S. HESS and L. WALDMANN, Z. Naturforsch. **25 a**, 1367 [1970]; **26 a**, 1057 [1971].

[12] V. G. COOPER, A. D. MAY, E. H. HARA, and H. F. P. KNAAP, Phys. Lett. **27 A**, 52 [1968].

[13] S. HESS, Z. Naturforsch. **25 a**, 350 [1970]; Springer Tracts Mod. Phys. **54**, 136 [1970].

[14] In deriving Eq. (6) the collisional coupling between the translational and rotational heat fluxes has been neglected.